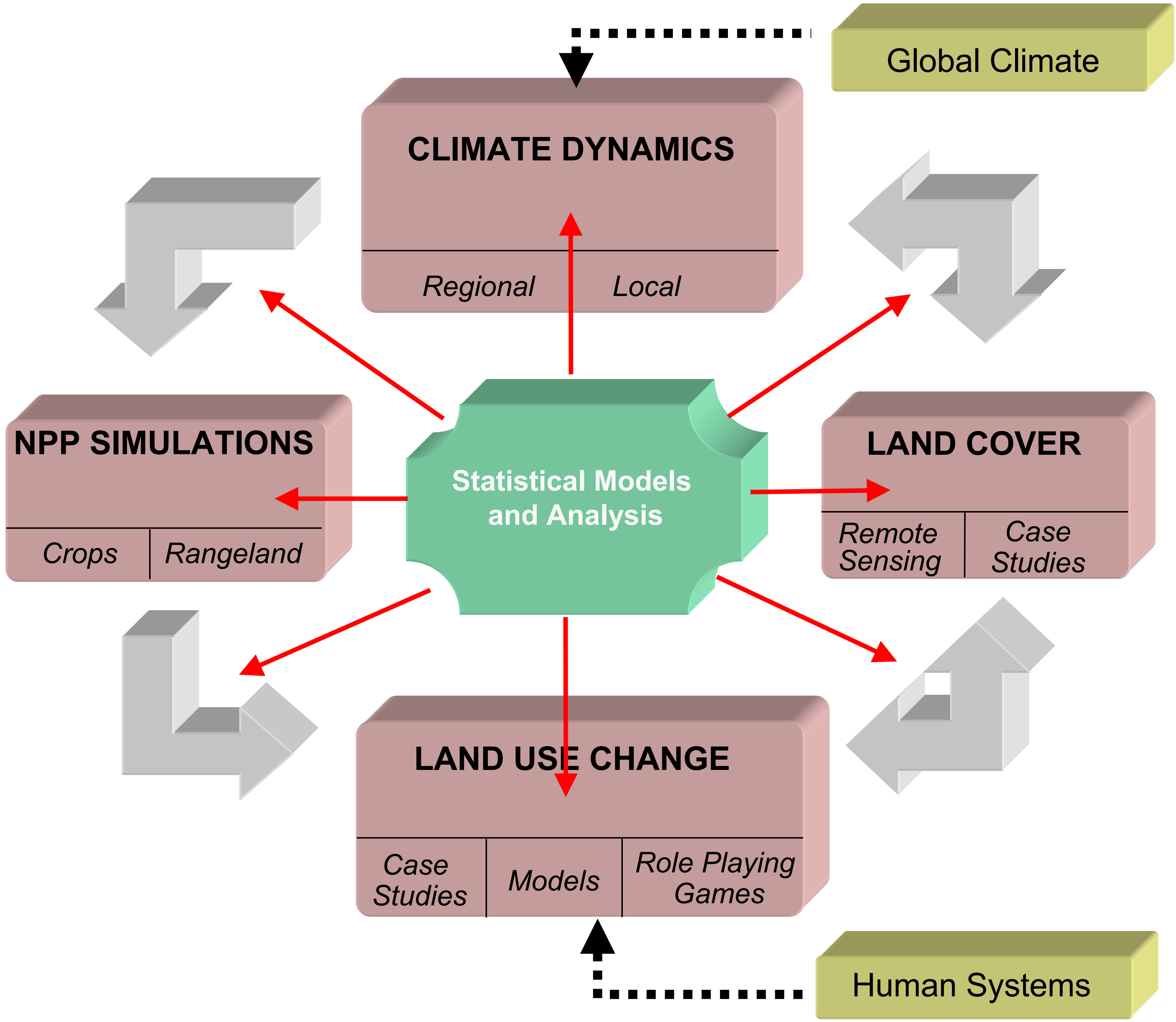
Uncertainty

CLIP Workshop

Sep 19-21


Global Climate
CLIMATE DYNAMICS
Regional
Local
NPP SIMULATIONS
Crops
Rangeland
Statistical Models and Analysis
LAND COVER
Remote Sensing
Case Studies
LAND USE CHANGE
Case Studies
Models
Role Playing Games
Human Systems

Three kinds of variability

- Planned, systematic variability (that we want)
- Random variability (that we have to live with)
- Unplanned, systematic variability (that destroys the validity of our conclusions)

Planning Experiments

1. **Overall research plan**
2. **Statement of problem/objective**
   – What is the experiment intended to do?
   – Sound questions go along way toward solution.
3. **Limitations (costs, resources):**
   – Do not spend more than 30 % of your budget on the first set of experiments!
4. **Choose measurements to make response**
   – Are variables measurable?
   – What sort of response is expected?
   – How accurately can response be measured?
   – **Reliability** (repeatability)
   – **Validity** (relevance of data/response to the purpose of the study)

Planning an Experiment (cont'd)

5. Choose conditions to study
   – What inputs may affect response?
   – What inputs are of interest?
   – Are factors to be held constant?
   – Varied at specific levels?
6. Choose experimental plan
   – How large a difference in the response is important?
   – How much variation is present?
   – What costs and resources are available?
   – What is the timing of the experiment?
   – Randomization

Planning an Experiment (cont'd)

7. Perform the experiment
   - Make sure the design can be implemented.
   - Is a trail run necessary?
   - Record deviations from planned experiment.
8. Analyze the data
   - Do the model assumptions hold?
   - On the basis of the model decide the number of replications.
9. Draw conclusions and make recommendations.
   - Conclusions should refer back to the stated objectives of the experiment.
   - Confirmation or follow-up experiment

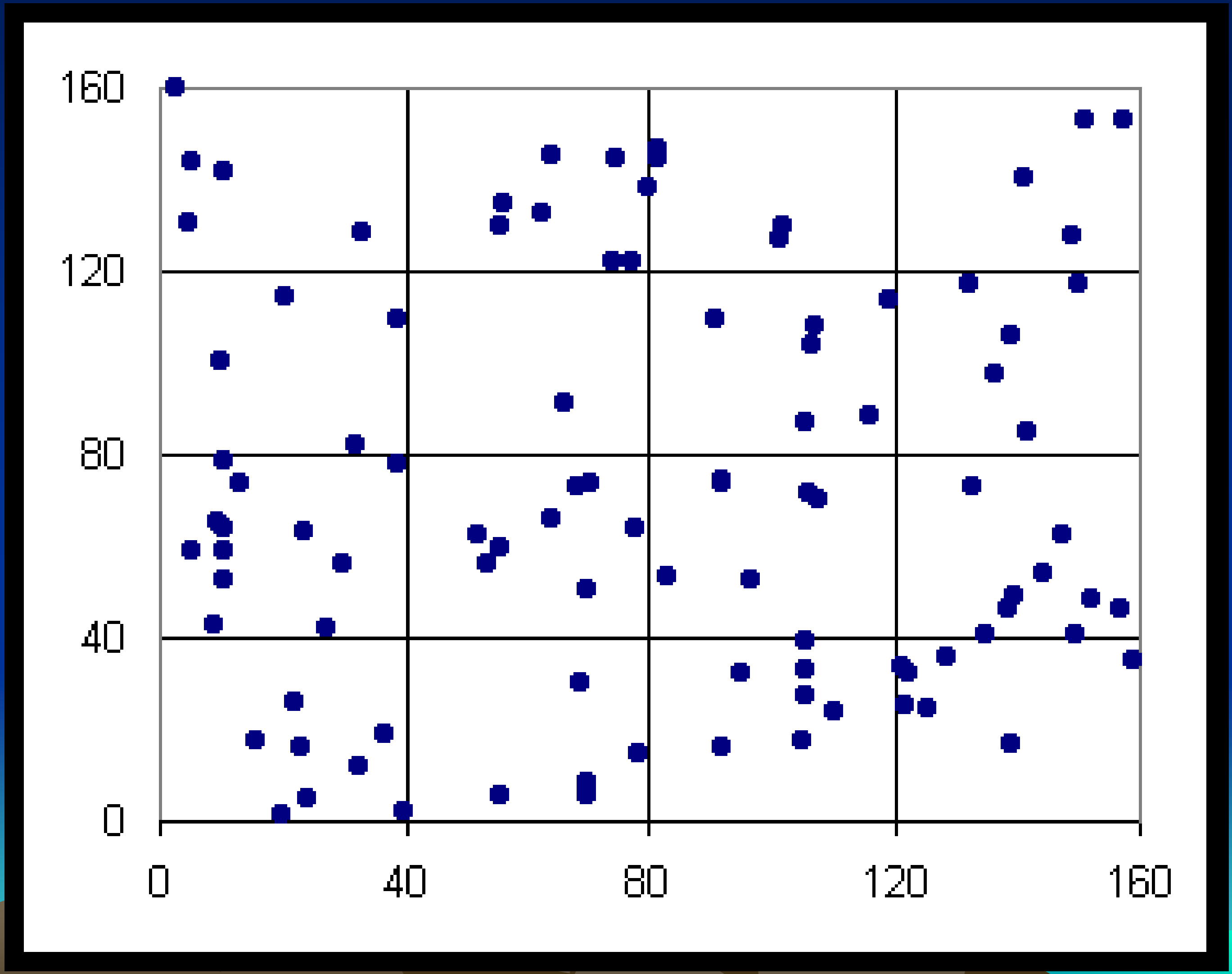
Priorities

- Specific questions
  - Priorities of the different questions/goals
  - Identification of influential factors
  - Characteristics to be measured
  - Experimental procedures (data collection, simulation)
  - Controls to be used
- Practical issues
  - Types of data/models
  - Replication
  - Resources

Random Spatial Patterns

Example 1

Random Spatial Patterns



Random Spatial Patterns

Independent placement of 400 points in a square 320 by 320 region.

The Figure shows the result for a 160 by 160 quadrant of this regions.

Model the number of points in each 40 by 40 square.

Approximation by a Poisson distribution model

x	0	1	2	3	4	5	6	7	8	9	10	11	12
obs	0	1	0	0	3	4	1	2	3	1	0	0	1
freq	0	.06	0	0	.19	.25	.06	.13	.19	.06	0	0	.06
P	0	.01	.04	.08	.12	.15	.16	.14	.11	.07	.05	.03	.01

Observations: mean = 6.125, standard deviation = 2.58

Poisson model: mean = 6.25, standard deviation = 2.50

Forecasting

Example 2

Time series analysis example

Quarterly unemployment rate (1948-2002) of men and women 20 years and older: $\{(R_{t,1}, R_{t,2})\}_{t=1}^{220}$

$$
\begin{aligned}
R_{t,1} &= \text{men's unemployment rate in quarter } t, \\
R_{t,2} &= \text{women's unemployment rate in quarter } t.
\end{aligned}
$$

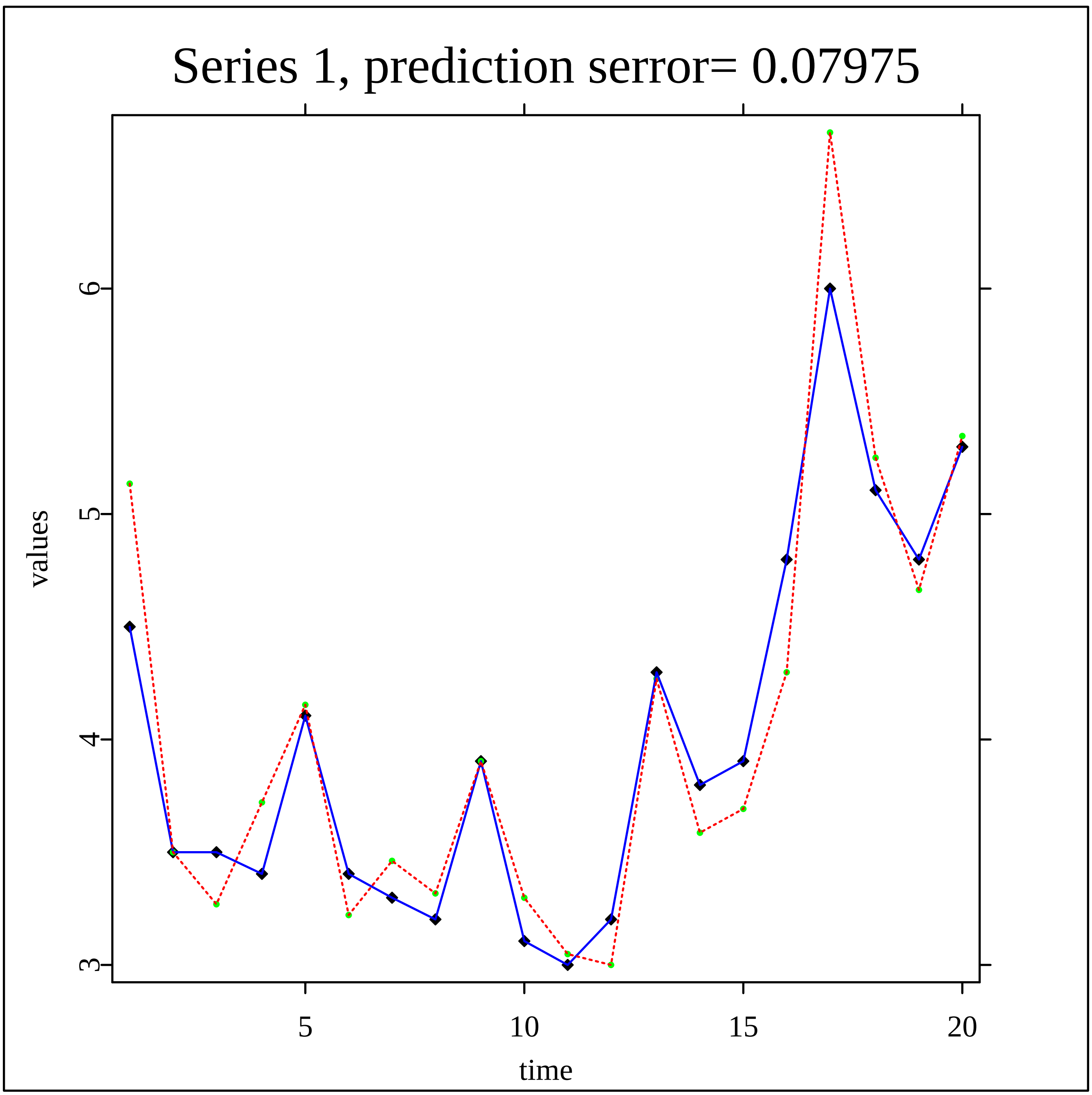
Vector autoregression model

forecast rates at time t based on rates in the past

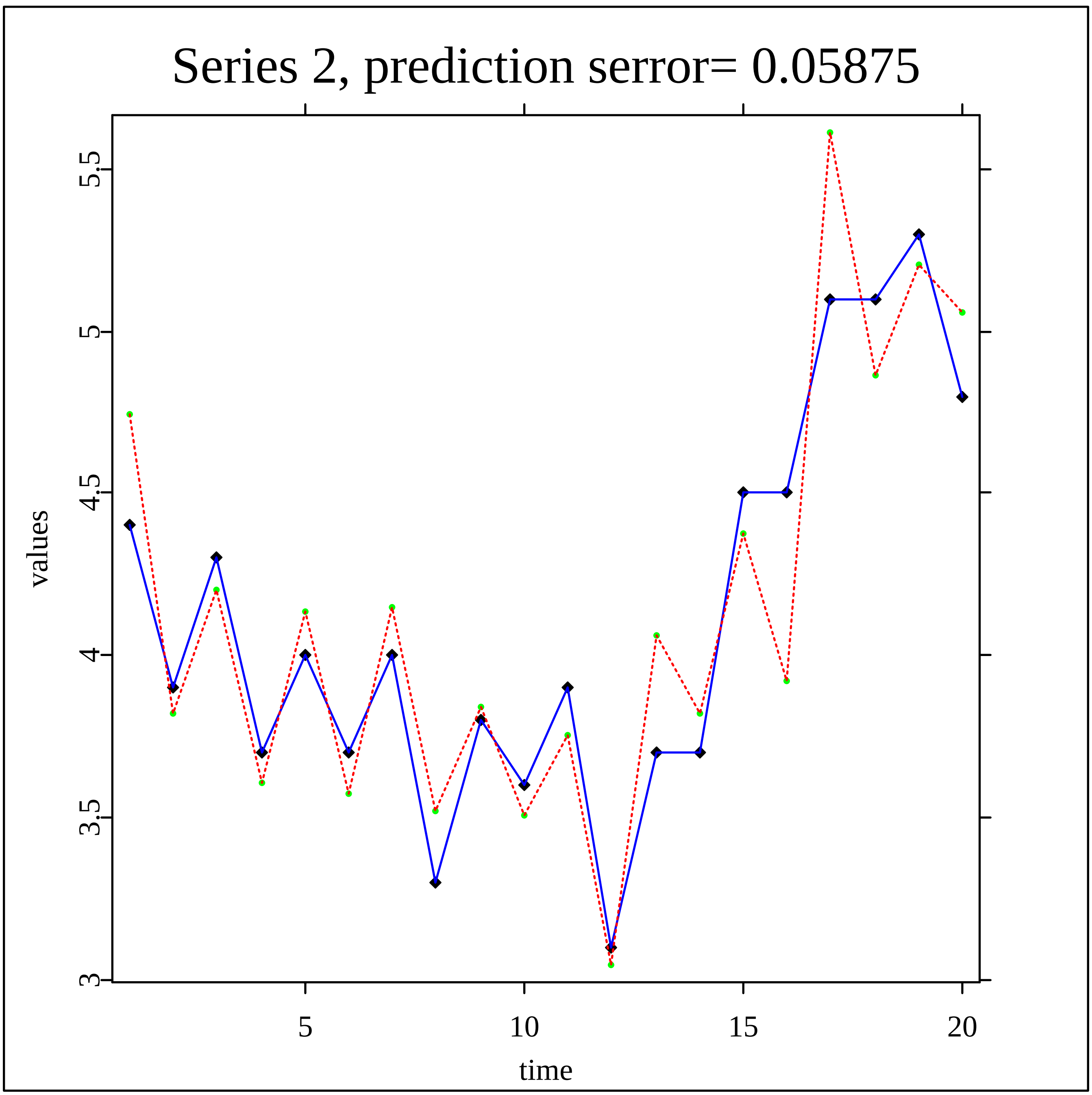
$$\hat{R}_{t,1} = f_1(R_{t-1,1}, R_{t-1,2}, R_{t-2,1}, R_{t-2,2}, R_{t-3,1}, R_{t-3,2}, ...,),$$
$$\hat{R}_{t,2} = f_2(R_{t-1,1}, R_{t-1,2}, R_{t-2,1}, R_{t-2,2}, R_{t-3,1}, R_{t-3,2}, ...,).$$

Rates of 1948-1997 ($t = 1, 2, ..., 200$) used to forecast rates of 1998-2002 ($t = 201, ..., 220$) using seasonal additive nonlinear vector autoregression model (SANVAR)

Forecasts of the US quarterly unemployment rate from 1998 to 2002 for men.



Forecasts of the US quarterly unemployment rate from 1998 to 2002 for women.



Risk Assessment Models

Example 3

Probabilistic Model for Finite Dam

Other uses for the model: storage model, flow system, queueing system, computer network etc.

blue Consider a reservoir with finite capacity c. Let

- Z_n = reservoir storage at time n;
- X_{n+1} = the water flowed in during the period $(n, n+1]$;
- due to possible overflow, the actual input during $(n, n+1]$ is

$$\eta_{n+1} = \min\{X_{n+1}, c - Z_n\};$$

- ξ_{n+1} = output during $(n, n+1]$.

Then at time $n+1$, the reservoir storage is

$$Z_{n+1} = \Big(Z_n + \eta_{n+1} - \xi_{n+1}\Big)^+.$$

Concern: The probability that the reservoir becomes empty (or below certain level):

$$P\Big\{\exists\ n > 1 \ \text{ such that } \ Z_n = 0\Big\}?$$

A model for insurance risk

Consider an insurance company with initial capital u and assume

- claims arrive at times $T_1 < T_2 < \dots$ with claim sizes $X_1, X_2, \dots;$
- the total amount $X(t)$ of claims during $(0, t]$ is a compound Poisson process

$$X(t) = \sum_{i=1}^{N(t)} X_i,$$

  where $N(t) = \#$ of claims in $(0, t]$;

- the company receives premiums at a constant rate β;

- the risk process is $Z(t) = u + \beta t - X(t), \quad t \geq 0.$

  Concern: ruin probability:

$$P\Big\{Z(t) < 0 \ \text{ for some } \ t > 0\Big\}$$

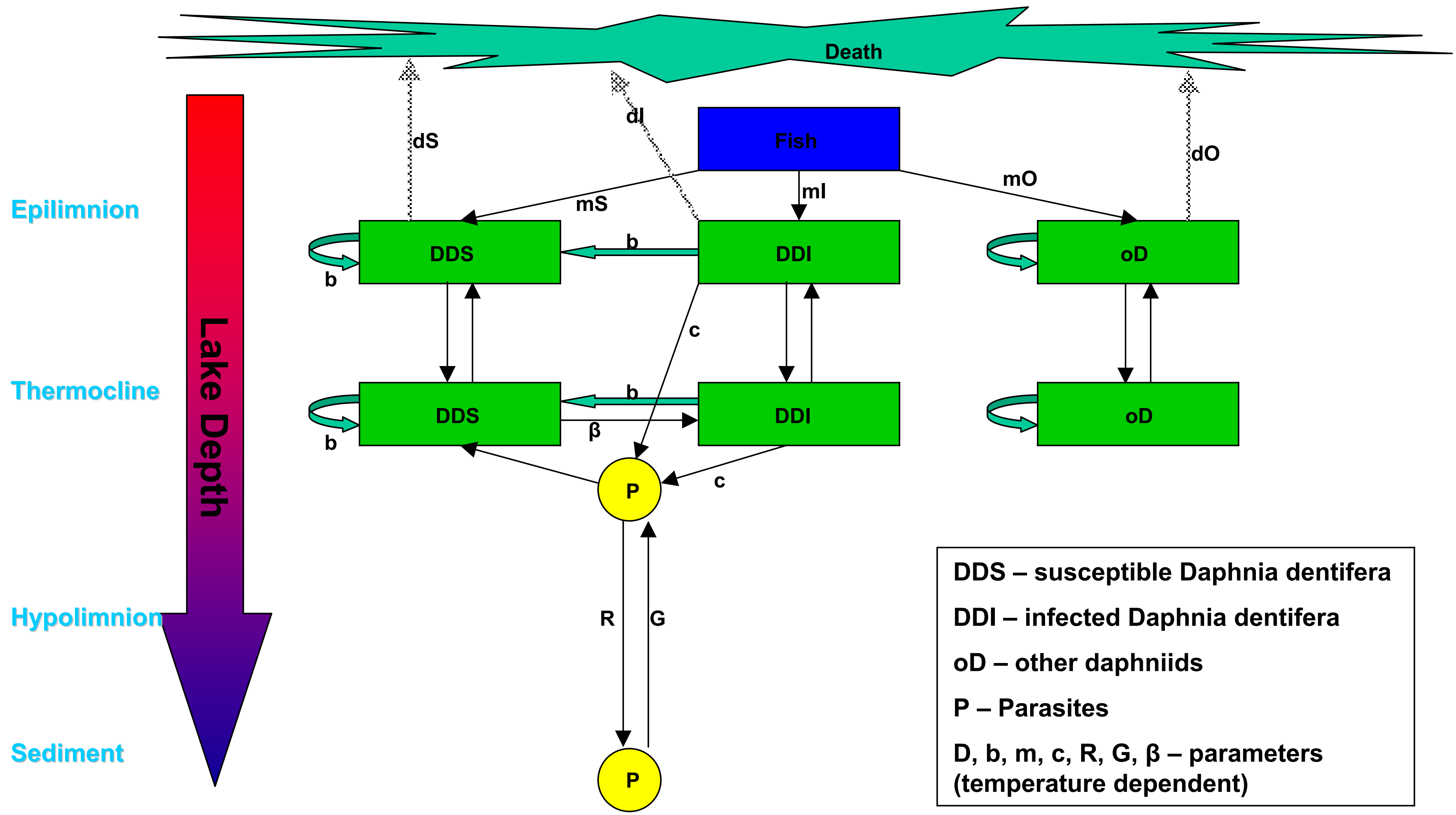
  and the ruin time $T = \inf\Big\{t > 0 : Z(t) < 0\Big\}.$

There are probabilistic and statistical tools to analyze these models based on various assumptions.

The Value of Simulations

Example 4

Daphnia System (Fall)



DDS – susceptible Daphnia dentifera

DDI – infected Daphnia dentifera

oD – other daphniids

P – Parasites

D, b, m, c, R, G, β – parameters (temperature dependent)

Daphnia Model

susceptibles

birth

carrying capacity

transmission

$$\frac{dS}{dt} = b(S+I)\left(1 - \frac{S+I}{K}\right) - \beta S Z_w - m_S S$$

mortality

infected

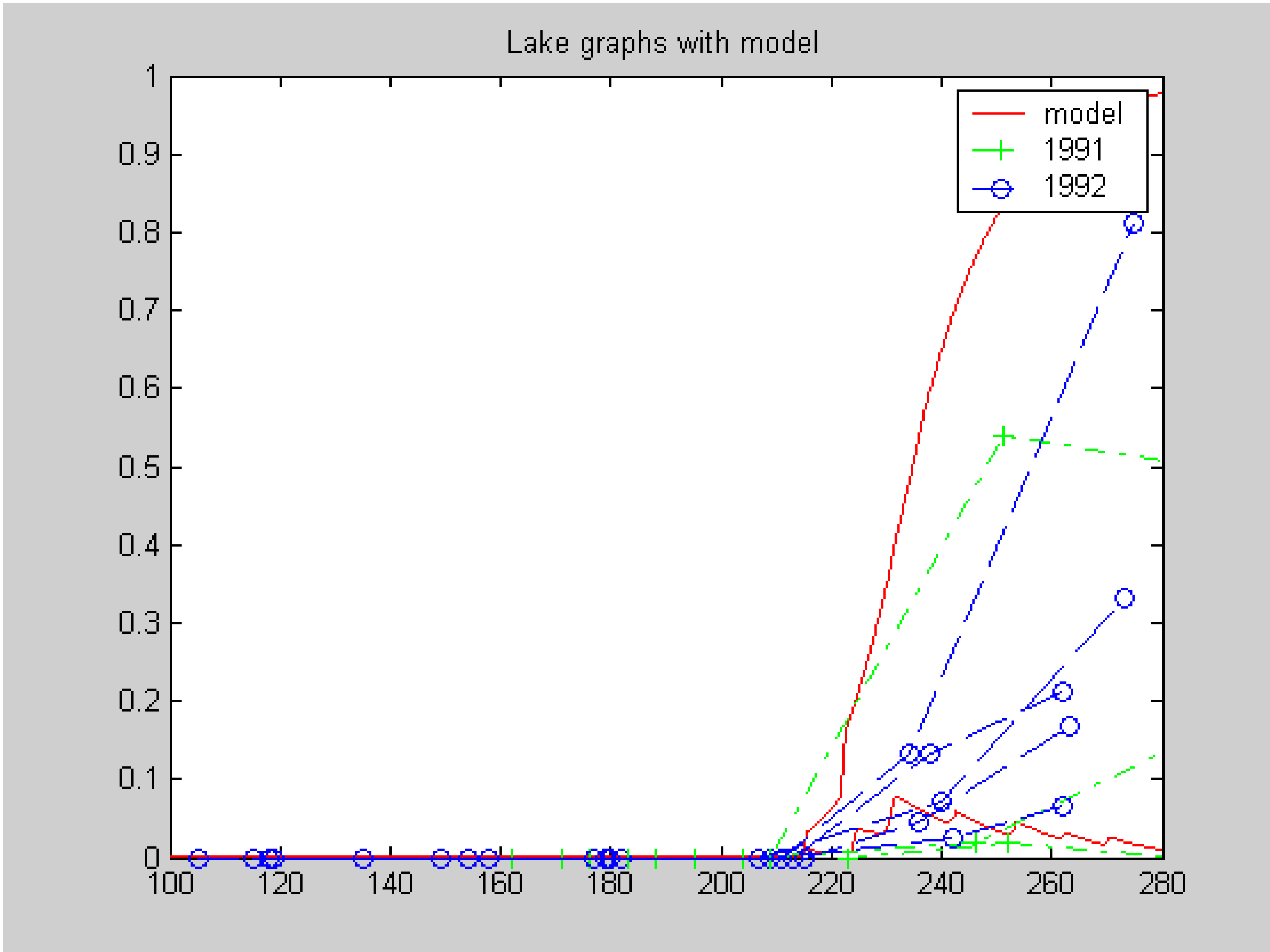
$$\frac{dI}{dt} = \beta S Z_w - m_I I$$

$$\frac{dZ_w}{dt} = cIf + G(Z_s) - \frac{Z_w}{R}$$

sinking

parasites

resuspension



Lake graphs with model
model
1991
1992
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
120
140
160
180
200
220
240
260
280

Inputs - Outputs

- Sensitivity analysis: what is the rate of change in the output when a single input is varied by a small amount while other inputs are held constant
- Scenario analysis: many inputs are varied at the same time
- Monte Carlo simulations: randomly choose an input -> probability distribution for the output

Scenarios and Impacts

Structure of the “impact space”?

- Which scenarios lead to similar outputs?
- How “close” are these outputs?

Multiple Testing

- Want to test null hypotheses H_{0i} versus alternative hypotheses H_{1i}, $i = 1, \ldots, m$.
- If we test each of the m hypotheses separately at level α, then the probability of making at least one error can be much larger than α.
- For example, if we conduct $m = 10$ independent tests and $\alpha = 0.05$, then the probability that at least one p-value will be less than α is 0.401. For $m = 100$ and $\alpha = 0.05$ the probability is 0.994.
- To fix this problem, some sort of adjustment of p-values or α-levels is made.

Example: Bonferroni Method

- Instead of testing each hypothesis at level α, test each at level α/m. Then the probability of at least one false rejection is bounded by α.
  For example, if $m = 8$ and $\alpha = 0.05$, test each hypothesis at level $0.05/8 = 0.00625$.

- What is wrong with the Bonferroni procedure?
  Example: All m null hypotheses specify $N(0, 1)$ distribution. All alternative hypotheses specify $N(\mu, 1)$ distribution, with $\mu \neq 0$. Let $n = 4$, and $\alpha = 0.05$.
  - If $m = 1$ (no multiple testing), the power is 0.516.
  - If $m = 5000$, the power is 0.00782.

R. Katz' Recommendations

- Assess uncertainty in individual model components separately
- Systematic treatment of uncertainty for less complex models (propagation of uncertainty)
- Statistical theory
  - Statistical downscaling
  - Extreme events
  - Bayesian approach (incorporate subjective knowledge)

Rubin’s Commandments

1. You have to make assumptions.
2. You have to know your assumptions.
3. You should not believe your assumptions.
4. You have to know the consequences of your assumptions.